Rekha V.V.I. Questions for 2022 Examination

Answers of below mentioned questions are present in your Rekha Examination Guide Part–II Mathematics – III (Hons.)

1. (a) State and Prove Leibnitz's theorem to find the nth derivative of a product of two functions of x 7

(b) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. 9

(c) If $y = \sin(m \sin^{-1} x)$ prove that :
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$ 10

2. (a) State and prove Euler's theorem on homogeneous functions of two independent variables. 18

(b) If $u = \sin^{-1}\dfrac{x^2+y^2}{x+y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$ 23

(c) If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$ 23

(d) Expand $e^{\sin x}$ as far as the term involving x^4. 16

3. (a) Prove that :

$$\frac{1}{p^2} = u^2 + \left(\frac{\partial u}{\partial \theta}\right)^2$$ 31

(b) Find the pedal equation of the curve : $r^m = a^m \cos m\theta$ 36

(c) Show that in the exponential curve $y = be^{x/a}$ the subtangent is of constant length and subnormal varies as the square of the ordinate. 31

4. (a) Prove that: $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$
Where the symbols have their' usual meaning. 26

(b) Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ may touch the curve $x^m y^n = a^{m+n}$. 30

5. (a) Prove that the radius of curvature of the curve y = f (x) is given by

$$P = \frac{\left(1+y_1^2\right)^{3/2}}{y^2}$$ 32

(b) Find the radius of curvature for the cartesian curve y = f (x). 32

6. Evaluate any two of the following :

(a) $\int \frac{dx}{(x-\alpha)(\beta-x)}$ 45

(b) $\frac{xdx}{(1 \quad x)(1 \quad x^2)}$ 44

(c) $\int \frac{dx}{4+5\sin x}$ 45

(d) $\int \frac{dx}{1+x^3}$ 44

(e) $\int \frac{\cos\ (\log x)}{x} dx$ 47

7. Evaluate any two of the following :

(a) $\int_0^1 tan^{-1} xdx$ 52

(b) $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ 51

(c) $\int_0^{\pi/2} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}}$ 50

(d) $\int_0^{\pi/2}$ log sin x dx 52

(e) $\int_0^{\pi/4} \sqrt{\tan x}\ dx$ 53

(f) $\int_0^1 \frac{\log\ (1+x)}{1+x^2} dx$ 51

8. (a) Find the area of the loop of the curve $ay^2 = x^2 (a - x)$ 59

(b) Find the area of the cardioid $r = a (1 + \cos\theta)$ 63

(c) Find the area included between the curve $y^2 (a - x) = x^3$ and its asymptote. 61

(d) Find the area of the loop of the curve $ay^2 = x^2 (a-x)$ 59

9. (a) Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$. 66

(b) Find the length of the arc of the curve $\frac{2a}{r} = 1+\cos\theta$ from $\theta =$

0 to $\theta = \pi/2$. 64

(c) Find the volume of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line. 73

10. (a) Obtain Lagrange's condition for Maxima or Minima of functions of two independent variables. 38

11. (a) Prove that a monotonic increasing sequence bounded above converges to least upper bound of the set of its terms : 85

(b) Show that: $\lim_{n\to\infty} n^{1/n} = 1$ 112

(c) Show that every convergent sequence is bounded. 84

12. (a) State and prove De Morgan and Bertrand's test. 109

(b) Test the convergency of the series

$$1+\frac{2^2}{3^2}+\frac{2^2}{3^2}\frac{4^2}{5^2}+\frac{2^2}{3^2}.\frac{4^2}{5^2}.\frac{6^2}{7^2}+.....$$ 106

13. (a) Test the convergence of the series $1+\frac{2^p}{\lfloor 2}+\frac{3^p}{\lfloor 3}+\frac{4^p}{\lfloor 4}+....$ 101

(b) State and prove D'Alembert's Ratio Test. 96

14. (a) Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{(\log n)^p}$ 102

(b) Define absolute convergence and conditional convergence of an infinite series of real numbers. Prove that every absolultely convergent series is convergent but not convergely. 107

15. (a) State and prove Cauchy's condensation test. 99

(b) Test the convergence of the series whose nth term is

$\frac{1}{(n\log n)^p}$. 103

MATHS. – 3 (Hons.) (2021)

1. (a) State $\alpha - \delta$ definition of limit of a function. Prove that every differentiable function is continuous.
   (b) If $y = (\sin^{-1} x)^2$, prove that– $(1-x^2)\, y_{n+2} + (2n+1)\, xy_{n+1} - n^2 y_n = 0$
2. (a) State and prove Maclaurin's theorem.
   (b) If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \sin 2u$.
3. (a) Find the condition that the line.
   $x \cos\alpha + y \sin\alpha = p$ should touch the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$.
   (b) Show that in any curve $\frac{\text{sub normal}}{\text{sub tangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$
4. (a) Find the radius of curvature in pedal form.
   (b) Prove that for a curve given by $r^2 = a^2 \cos 2\theta$, we have $e = \frac{a^2}{3r}$.
5. Evaluate any two of the following :
   (a) $\int \frac{x^2+1}{x(x^2-1)}dx$ (b) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (c) $\int(\sqrt{\tan x} + \sqrt{\cot x})\,dx$ (d) $\int \frac{dx}{5+3\cos x}$
6. Evaluate any two of the following :
   (a) $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}dx$ (b) $\int_0^{\pi/2} \cos^n x \cos nx\, dx$
   (c) $\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ (d) $\int_0^\pi \frac{dx}{a+b\cos x} (a>b>0)$
7. (a) If $B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$, Prove that $B(m, n) = B(n, m)$
   (b) Find the area of a loop of the curve $r^2 = a^2 \cos 2\theta$.
8. (a) Find the perimeter of the loop of the curve $9ay^2 = (x-2a)(x-5a)^2$
   (b) Find the perimeter of the cardioid : $r = a(1-\cos\theta)$
9. Find the surface area of a right circular cone whose semi-vertical angle is α, height h ane base is circular of radius a. Also find the volume of the cone.
10. (a) State and prove Cauchy's general principle of convergence for a sequence.
    (b) Prove that $\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right) = 0$
11. (a) State and prove comparison test to examine the convergence of an infinite series of non-negative terms.
    (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$
12. (a) State and prove Gauss's test for the convergence of an infinite series.
    (b) In an absolutely convergent series, show that the series formed by its positive terms alone is convergent and the series formed by its negative terms alone is convergent.

MATHS. – 3 (Hons.) (2020)

Answer any six questions

1. (a) State and Prove Leibnitz's theorem to find the nth derivative of a product of two functions of x 7

   (b) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)\, y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$ 9

2. (a) State and prove Euler's theorem on homogeneous functions of two independent variables. 18

   (b) If $u = \sin^{-1} \dfrac{x^2 + y^2}{x + y}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$ 23

3. (a) Prove that: $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$

   Where the symbols have their' usual meaning. 31

   (b) Find the condition that the line $x \cos\alpha + y \sin\alpha = p$ may touch the curve $x^m y^n = a^{m+n}$. 26

4. (a) Prove that the radius of curvature of the curve $y = f(x)$ is given by

   $$P = \frac{\left(1 + y_1^2\right)^{3/2}}{y^2}$$ 32

   (b) Find the radius of curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point where $y = x$ cuts it.

5. Evaluate any two of the following :

   (a) $\displaystyle\int \frac{dx}{(x-\alpha)(\beta - x)}$ 45

   (b) $\dfrac{xdx}{(1 \quad x)(1 \quad x^2)}$ 44

   (c) $\displaystyle\int \frac{dx}{4 + 5\sin x}$ 45

   (d) $\int e^{-x} \cos^2 x dx$

6. Evaluate any two of the following :

   (a) $\int_0^1 \tan^{-1} x dx$ 52

   (b) $\displaystyle\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ 51

(c) $\int_0^{\pi/2} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$ 50

(d) $\int_0^{\pi/2} \log \sin x \, dx$ 52

7. (a) Find the area of the loop of the curve $ay^2 = x^2(a - x)$ 59
(b) Find the area of the cardioid $r = a(1 + \cos\theta)$ 63

8. (a) Find the whole length of the loop of the curve $3ay^2 = x(x - a)^2$. 66

(b) Find the length of the arc of the curve $\frac{2a}{r} = 1+\cos\theta$ from $\theta = 0$ to $\theta = \pi/2$. 64

9. (a) Obtain Lagrange's condition for Maxima or Minima of functions of two independent variables. 38

(b) Prove that $\beta_{m,n} = \frac{\lceil m \lceil n}{\lceil m+n}$

10. (a) Prove that a monotonic increasing sequence bounded above converges to least upper bound of the set of its terms : 85

(b) Show that: $\lim_{n\to\infty} n^{1/n} = 1$ 112

11. (a) State and prove De Morgan and Bertrand's test. 109
(b) Test the convergency of the series $1+\frac{2^2}{3^2}+\frac{2^2}{3^2}\frac{4^2}{5^2}+\frac{2^2}{3^2}.\frac{4^2}{5^2}.\frac{6^2}{7^2}+.....$ 106

12. (a) Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{(\log n)^p}$ 102

(b) Define absolute convergence and conditional convergence of an infinite series of real numbers. Prove that every absolultely convergent series is convergent but not convergely. 107

MATHS. - 3 (Hons.) (2019)

Answer any six questions

1. (a) State $\in - \delta$ definition of limit of a function. Prove that every differentiable function is continuous. 5

(b) If $y = \sin(m \sin^{-1} x)$ prove that :
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)\, y_n = 0$ 10

2. (a) State and prove Maclaurin's theorem. 14

(b) If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$ 23

3. (a) Prove that :

$$\frac{1}{p^2} = u^2 + \left(\frac{\partial u}{\partial \theta}\right)^2$$ 31

(b) Find the pedal equation of the curve :
$r^m = a^m \cos m\theta$ 36

4. (a) Find the radius of curvature for the cartesian curve y = f (x). 32

(b) For any curve, prove the formula,

$$e = \frac{r}{\sin\phi(1+\frac{d\phi}{d\theta})}, \text{ where } \tan\phi = r\frac{d\theta}{dr}.$$

5. Evaluate any two of the following :

(a) $\displaystyle\int \frac{dx}{1+x^3}$ – 44 (b) $\displaystyle\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ – 42

(c) $\displaystyle\int \frac{\cos\ (\log x)}{x} dx$ – 47 (d) $\displaystyle\int \frac{dx}{5+4\cos x}$ –46

6. Evaluate any two of the following :

(a) $\displaystyle\int_0^{\pi/4} \sqrt{\tan x}\ dx$ – 53 (b) $\displaystyle\int_0^{\pi/4} \cos^n\ x \cos n\ x\ dx$ – 58

(c) $\displaystyle\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ – 48 (d) $\displaystyle\int_0^1 \frac{\log\ (1+x)}{1+x^2} dx$ – 51

7. (a) Find the area included between the curve $y^2 (a - x) = x^3$ and its asymptote. 61

(b) If $B(m, n) = \displaystyle\int_0^1 x^{m-1}(1-x)^{n-1} dx$. Prove that

$B(m, n) = B(n, m)$ 75

8. (a) Find the perimeter of the loop of the curve :
$9ay^2 = (x - 2a)(x - 5a)^2$ 67

(b) Find the perimeter of the cardioid :

$r = a(1 - \cos\theta)$

9. Find the surface area of a right circular cone whose semi-vertical angle is α, height h and base is circular of radius a. Also find the volume of the cone. 74

10. (a) State and prove Cauchy's general principle of convergence for a sequence. 88

(b) Prove that : $\lim_{n\to\infty} \frac{1}{n}(1+\frac{1}{2}+\frac{1}{3}+--+\frac{1}{n}) = 0$

11. (a) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

(b) State and prove D'Alembert's Ratio Test. 96

12. (a) State and prove Raabe's test for the convergence of an infinite series. 98

(b) In an absolutely convergent series, show that the series formed by its positive terms alone is convergent and the series formed by its negative terms alone is convergent.

MATHEMATICS - 3 (Hons.) (2018)

Answer any six questions.

1. (a) State and prove Leibnitz's theorem to find the nth derivative of a product of two functions of x. 7

(b) If $y = e^{a\sin^{-1}x}$, prove that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$ 9

2. (a) State and prove Euler's theorem on homogeneous functions of two independent variables. 18

(b) Expand $e^{\sin x}$ as far as the term involving x^4. 16

3. (a) Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ should touch the curve $x^m y^n = a^{m+n}$ 26

(b) Show that in the exponential curve $y = be^{x/a}$ the subtangent is of constant length and subnormal varies as the square of the ordinate. 30

4. (a) Find the radius of curvature for the pedal curve p = f (r). 33

(b) For the curve $r^m = a^m \cos m\theta$, prove that $e = \frac{a^m}{(m+1)r^{m-1}}$ 36

5. Evaluate any two of the following :

(a) $\int \frac{dx}{(x-3)\sqrt{x+1}}$ – 43 (b) $\int \frac{dx}{5+3\cos x}$ – 46

(c) $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ – 47 (d) $\int x^p \sin q^x \, dx$ –54

6. Evaluate any two of the following :

(a) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ –50 (b) $\int_0^{\pi} \frac{dx}{a+b\cos x}(a>b>0)$ –49

(c) $\int_0^{\pi/2} \log \sin x dx$ – 52 (d) $\int_0^{\pi/2} \cos^n x dx$ – 57

7. (a) Find the area of the loop of the curve $ay^2 = x^2(a-x)$ 59

(b) Find the area of the cardioid $r = a(1+\cos\theta)$. 63

8. (a) Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$. 66

(b) Find the volume of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line. 73

9. (a) Obtain Lagrange's condition for Maxima or Minima of functions of two independent variables. 38

(b) Prove that $B(m,n) = \frac{\overline{|(m)}\,\overline{|(n)}}{\overline{|(m+n)}}$

10. (a) Show that every convergent sequence is bounded. 84

(b) Show that : $\lim_{x\to\infty} n^{1/n} = 1$ 112

11. (a) State and prove Gauss' test for the convergence of a series. 91

(b) Test the convergence of the series $1+\frac{2^p}{\lfloor 2}+\frac{3^p}{\lfloor 3}+\frac{4^p}{\lfloor 4}+....$ 101

12. (a) State and prove Cauchy's condensation test. 99

(b) Test the convergence of the series whose nth term is $\frac{1}{(n\log n)^p}$. 103

Rekha V.V.I. Questions for 2022 Examination

Answers of below mentioned questions are present in your Rekha Examination Guide Part–II Mathematics – IV (Hons.)

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1. (a) Show that every finite integral domain is a field. **10**

   (b) The necessary and sufficient condition for a non-empty subset S of a ring R to be subring of R are:

   (i) $a \in S, b \in S \Rightarrow a - b \in S$ (ii) $a \in S, b \in S \Rightarrow a\,b \in S$ **12**

   (c) Show that a ring R is without zero divisor if and only if the cancellation laws hold in R. **7**

2. Solve any two of the following differential equations :

   (a) $\frac{dy}{dx} = \sin(x + y)$ **– 15**

   (b) $\frac{dy}{dx} = x^3y^3 - xy$ **– 23**

   (c) $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ **– 43**

   (d) $y \sin 2x\, dx - (1+y^2 + \cos^2 x)\, dy = 0$ **– 24**

   (e) $\frac{dy}{dx} \quad \frac{y}{x} \quad \tan\frac{y}{x}$ **– 15**

   (f) $x\frac{dy}{dx} + y = y^2 \log x$ **– 20**

   (g) $(1 + xy)\, y\, dx + (1 - xy)\, x\, dy = 0$ **– 45**

3. Solve any two of the following differential equations :

   (a) $p^2 + 2xp - 3x^2 = 0$ **– 64**

   (b) $y = (1 + p)\, x + ap^2$ **– 61**

   (c) $y = p^2y + 2px$ **– 59**

   (d) $y = 2px + y^2 p^3$ **– 62**

   (e) $p^2 - py + x = 0$ **– 60**

4. Solve any two of the following differential equations :

   (a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$ **– 32**

   (b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x \cos x$ **– 33**

   (c) $\frac{d^2y}{dx^2} + y = x^3 + e^x \sin x$ **– 36**

(d) $\frac{d^2y}{dx^2} + 4y = \sec 2x$ – 31

(e) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$ – 35

(f) $\frac{d^2y}{dx^2} + a^2y = \sec ax$ – 38

5. (a) Find the orthogonal trajectory of the family of circles $x^2 + y^2 = 2ax$ each of which touches the y-axis at the origin. 53

(b) Find the orthogonal trajectory of family of curve : $x^{2/3} + y^{2/3} = a^{2/3}$ 40

(c) Using the method of variation of parameters solve the differential equation : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 ex$ 30

(d) Using the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2} + n^2y = \sec nx.$ 51

6. Solve any two of the following differential equations :

(a) $x^2 (y-z) p + y^2 (z - x) q = z^2 (x - y)$ 70

(b) $y^2p + x^2q = z^2y^2x^2$ 73

(c) $(y + z) p + (z + x) q = x + y$ 69

(d) Apply charpit method to solve the differential equation $(p^2 + q^2) y = qz$ for complete integral. 74

(e) $\frac{dx}{dt} + 4x + 3y = t, \quad \frac{dy}{dt} + 2x + 5y = e^t$ 49

(f) Using Charpit's method find the complete integral of the equation $p^2x + q^2y = z$ 76

7. Prove the following recurrence formula for $J_n(x)$:

(a) $xJ_{n'}(x) = -n J_n(x) + x J_{n-1}(x)$ (b) $2J_{n'}(x) = J_{n-1}(x) - J_{n+1}(x)$ 80

8. Prove the following recurrence relations for the Legendre's polynomial $P_n(x)$: $(2n + 1)xP_n = (n + 1) P_{n+1} + nP_{n-1}$ 92

9. Apply Charpit's method to find the complete integral any one of the following :

(a) $x^2p^2 + y^2q^2 = z^2$ 74

(b) $p^2 + q^2 - 2px - 2qy + 1 = 0$ 77

10. Prove the following for Legendre polynomial $P_n(x)$: 92

(a) $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}(x^2-1)^n$

(b) $\int_{-1}^{1} P_n(x) P_m(x)\,dx = 0 \quad \text{if } m \neq n$

$= \frac{2}{2n+1} \text{ if } m = n$

11. Prove the following for Hermite polynomial :
(a) $H_{n'}(x) = 2_n H_{n-1}(x), n \geq 1$ (b) $2xH_n(x) = 2_n H_{n-1}(x) + H_{n+1}(x)$ 101

12. If n is a positive integer, show that
$J_{-n}(x) = (-1)^n J_n(x)$. 83
(b) Prove the following recurrence formula for $J_n(x)$:
$xJ_n = nJ_n - xJ_{n+1}$ 80

13. (a) State and prove second shifting theorem of Laplace transform. 107
(b) Find the Laplace transform of $e^{-2t}(3\cos 6t - 5\sin 6t)$ 121

14. (a) Find the Laplace transforms of $t^3 e^{-3t}$ 108
(b) Show that : $L(t\sin at) = \frac{2as}{(s^2+a^2)^2}$ 122

15. (a) State and Prove Convolution Theorem. 115
(b) Find the inverse transform of any one of the following :

(i) $\frac{1}{(s+1)(s^2+1)}$ – 112

(ii) $\frac{4s \quad 5}{(s \quad 1)^2 (s \quad 2)}$ – 111

(iii) $\frac{s^2 - 3s + 4}{s^3}$ – 115

16. Apply the method of Laplace transform to solve any one of the following differential equations :

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ where $y(0) = 0$ and $y'(0) = 1$. 125

MATHS. – 4 (Hons.) (2021)

Answer any six questions.

1. (a) Prove the following in a ring ?

   (i) $a(-b) = -(ab) = (-a)b$ (ii) $a(b-c) = ab - ac$ (iii) $(-a)(-b) = ab$

   (b) Show that every field is an integral domain.

2. Solve any two of the following :

   (i) $\frac{dy}{dx} = (x+y)^2$

   (ii) $(x^2 - y^2)\frac{dy}{dx} = 2xy$

   (iii) $x\frac{dy}{dx} + y = y^2 \log x$

   (iv) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$

3. Solve any two of the following :

   (i) $p^2 + 2py \cot x = y^2$

   (ii) $y = apx + bp^2,\ p = \frac{dy}{dx}$

   (iii) $y = xp + \frac{a}{p},\ p = \frac{dy}{dx}$

   (iv) Putting $x^2 = u$ and $y^2 = v$ reduce the equation $x^2(y - xp) = yp^2$ into Clairaut's form and hence solve it, where $p = \frac{dy}{dx}$

4. Solve any two of the following differential equations :

   (i) $\frac{d^2y}{dx^2} + y = \sin 2x$

   (ii) $\frac{d^2y}{dx^2} + 4y = x^3 + e^x + \sin x$

   (iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

   (iv) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x \cos x$

5. (a) Find the orthogonal trajectories of the family of cardioid $r = a(1 + \cos\theta)$

   (b) Using method of variation of parameters solve the following differential equation.

   $$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x$$

6. Solve any two of the following differential equations :

(i) $(mz - ny)\,p + (nx - lz)\,q = ly - mx$

(ii) $x\,(y^2 + z)\,p - y\,(x^2 + z)\,q = z\,(x^2 - y^2)$

(iii) $(x^2 - yz)\,p + (y^2 - zx)\,q = z^2 - xy$

(iv) $(y^2 + z^2 - x^2)\,p - 2xyq = -2xz$

$$\text{where } p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$$

7. Prove the following recurrence relations for the Legendre's polynomial $P_n(x)$:

(i) $nP_n(x) = (2n - 1)\,x\,P_{n-1}(x) - (n - 1)\,P_{n-2}(x)$

(ii) $nP_n(x) = n\,P'_n(x) - P'_{n-1}(x)$

8. Prove the following relations for $J_n(x)$:

(i) $2n\,J_n(x) = x\,\{J_{n-1}(x) + J_{n+1}(x)\}$

(ii) $\dfrac{d}{dx}\{x^{-n}J_n(x)\} = -\,x^{-n}\,J_{n+1}(x)$

9. (a) Show that

$$e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{\underline{|n}}\,H_n(x)$$

(b) $$\int_{-\infty}^{\infty} e^{-x^2}\,H_n(x)\,H_m(x)\,dx = \begin{cases} 0 & ,\text{if } m \neq a \\ \sqrt{\pi}\,2^n\,\underline{|n}, & \text{if } m = n \end{cases}$$

10. (a) Prove the first shifting formula of Laplace transformation.

(b) Find the Laplace transform of $\sin^3 2t$.

11. (a) Find the inverse transform of any one of the following :

(i) $\dfrac{s+2}{s^2 - 4s + 13}$

(ii) $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$

(b) If $L\{f(t)\} = \bar{f}(s)$, then show that

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{\bar{f}(s)\},\ \text{where } n = 1, 2, 3 \ldots.$$

12. Apply the method of Laplace transform to solve any one of the following differential equations.

(a) $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + x = e^t$ with $x = 2,\ \dfrac{dx}{dt} = 1$ at $t = 0$

(b) $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = 4t + e^{3t}$ when $y(0) = 1$ and $y'(0) = -1$

MATHS. – 4 (Hons.) (2020)

Answer any six questions

1. (a) Show that every finite integral domain is a field. 10

(b) The necessary and sufficient condition for a non-empty subset S of a ring R to be subring of R are:

(i) $a \in S, b \in S \Rightarrow a - b \in S$ (ii) $a \in S, b \in S \Rightarrow a\,b \in S$ 12

2. Solve any two of the following differential equations :

(a) $\frac{dy}{dx} = \sin(x + y)$ **– 15**

(b) $\frac{dy}{dx} = x^3y^3 - xy$ **– 23**

(c) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$ **– 43**

(d) $y \sin 2x\, dx - (1+y^2 + \cos^2 x)\, dy = 0$ **– 24**

3. Solve any two of the following differential equations :

(a) $p^2 + 2xp - 3x^2 = 0$ **– 64**

(b) $y = (1 + p)\, x + ap^2$ **– 61**

(c) $y = p^2y + 2px$ **– 59**

(d) $y = 2px + y^2 p^3$ **– 62**

4. Solve any two of the following differential equations :

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$ **– 32**

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x \cos x$ **– 33**

(c) $\frac{d^2y}{dx^2} + y = x^3 + e^x \sin x$ **– 36**

(d) $\frac{d^2y}{dx^2} + 4y = \sec 2x$ **– 31**

5. (a) Find the orthogonal trajectory of family of curve :

$x^{2/3} + y^{2/3} = a^{2/3}$ 53

(b) Using the method of variation of parameters solve the differential equation : $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2\, ex$ 40

6. Solve any two of the following differential equations :

(a) $x^2 (y-z)\, p + y^2 (z - x)\, q = z^2 (x - y)$ 70

(b) $y^2 p + x^2 q = z^2 y^2 x^2$ 73

(c) $(y + z)\, p + (z + x)\, q = x + y$ 69

(d) Apply charpit method to solve the differential equation $(p^2 + q^2)\, y = qz$ for complete integral. 74

7. Prove the following recurrence formula for $J_n(x)$:
(a) $xJ_{n'}(x) = -n\,J_n(x) + x\,J_{n-1}(x)$ (b) $2J_{n'}(x) = J_{n-1}(x) - J_{n+1}(x)$ **80**

8. Prove the following for Legendre polynomial $P_n(x)$: **92**

(a) $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}(x^2 - 1)^n$

(b) $\int_{-1}^{1} P_n(x)\,P_m(x)\,dx = 0 \quad \text{if } m \neq n$

$= \frac{2}{2n+1} \text{ if } m = n$

9. Prove the following for Hermite polynomial :
(a) $H_{n'}(x) = 2_n H_{n-1}(x),\ n \geq 1$ (b) $2xH_n(x) = 2_n H_{n-1}(x) + H_{n+1}(x)$ **101**

10. (a) State and prove second shifting theorem of Laplace transform. **107**
(b) Find the Laplace transform of $e^{-2t}(3\cos 6t - 5\sin 6t)$ **121**

11. (a) State and Prove Convolution Theorem. **115**
(b) Find the inverse transform of any one of the following :

(i) $\frac{s+1}{s^2+6s+25}$ (ii) $\frac{1}{(s+1)(s^2+1)}$ **112**

12. Apply the method of Laplace transform to solve any one of the following differential equations :

(a) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ where $y(0) = 0$ and $y'(0) = 1$. **125**

(b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$ where $y(0) = 0$ and $y'(0) = 1$.

MATHS. - 4 (Hons.) (2019)

Answer any six questions

1. (a) Show that a ring R is without zero divisor if and only if the cancellation laws hold in R. **7**
(b) Show that every field is an integral domain. **13**

2. Solve any two of the following differential equations :

(a) $(x-y)^2 \frac{dy}{dx} = a^2$ (b) $\frac{dy}{dx} \quad \frac{y}{x} \quad \tan\frac{y}{x}$ **– 15**

(c) $x\frac{dy}{dx} + y = y^2 \log x$ **– 20** (d) $(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$ **– 45**

3. Solve any two of the following differential equations :

(a) $p^2 + 2\,py\cot x = y^2$ 63

(b) $y = 2px + p^2$ 66

(c) $y = px + \sin^{-1} p$

(d) Putting $x^2 = u$ and $y^2 = v$, reduce the equation $x^2 (y - xp) = yp^2$ into Clairaut's form and hence solve it, where $p = \frac{dy}{dx}$ 58

4. Solve any two of the following differential equations :

(a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$ 35

(b) $\frac{d^2y}{dx^2} + 4y = \sin 3x + e^x + x^2$ 32

(c) $\frac{d^2y}{dx^2} + y = x^3 + e^x + \sin x$

(d) $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ 30

5. (a) Using method of variation of parameters, solve the following differential equation

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x$$

(b) Find the orthogonal trajectory of the family of circles $x^2 + y^2 = 2ax$ each of which touches the y-axis at the origin. 51

6. Solve any two of the following differential equations :

(a) $\frac{dx}{dt} + 4x + 3y = t, \quad \frac{dy}{dt} + 2x + 5y = e^t$ 49

(b) $x (y^2 + z) p - y (x^2 + z) q = z (x^2 - y^2)$ 71

(c) $x (y^2 + z) p - y (x^2 + z) q = z (x^2 - y^2)$ 71

(d) Using Charpit's method find the complete integral of the equation $p^2x + q^2y = z$ 76

7. Prove the following recurrence relations for the Legendre's polynomial $P_n (x)$:

(a) $(2n + 1)xP_n = (n + 1) P_{n+1} + nP_{n-1}$ 92

(b) $nP_n = x P'_n - P'_{n-1}$ 92

8. (a) Show that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^2}{\lfloor n} H_n(x)$ 102

(b) Show that :

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x) dx = \begin{cases} 0 & if \quad m \neq n \\ \sqrt{\pi}\ 2^n \ \underline{|n} & if \quad m = n \end{cases}$$ 103

9. If n is a positive integer, show that
$J_{-n}(x) = (-1)^n J_n(x)$. 83
(b) Prove the following recurrence formula for $J_n(x)$:
$$xJ_n = nJ_n - xJ_{n+1}$$ 80

10. (a) Prove the first shifting property of Laplace Transform. 107
(b) Find the Laplace transforms of $\sin^3 2t$. 119

11. (a) Find the inverse transform of anyone of the following :

(i) $\frac{4s \quad 5}{(s \quad 1)^2 (s \quad 2)}$ – 111 (ii) $\frac{s^2 - 3s + 4}{s^3}$ – 115

(b) State and prove Convolution theorem. 115

12. Apply the method of Laplace transform to solve anyone of the following differential equations :

(a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + X = e^t$ $with\ X = 2\frac{dx}{dt} = -1\ at\ t = 0$ 126

(b) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$ $with\ y = (0) = 1\ and\ y'(0) = -1$ 127

MATHEMATICS - 4 (Hons.) (2018)

Answer any six questions.

1. (a) Show that every finite integral domain is a field. 10
(b) Prove the following in a ring R :
(i) a (–b) = – (ab) = (–a) b
(ii) (–a) (–b) = ab
(iii) a (b – c) = ab – ac 6

2. Solve any two of the following :

(a) $\frac{dy}{dx} = \sin(x+y)$ – 15 (b) $(x^2 - y^2)\frac{dy}{dx} = 2xy$ – 42

(c) $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ – 43 (d) $\frac{dy}{dx} + 1 = e^{x-y}$ – 18

3. Solve any two of the following :

(a) $y = (1+p)x + ap^2$ – 61 (b) $y = (1+p)x + ap^2$ – 61

(c) $p^2 - py + x = 0$ – 60 (d) $y = 2px + y^2p^3$, where $p = \frac{dy}{dx}$ – 62

4. Solve any two of the following :

(a) $\frac{d^2y}{dx^2}+a^2y=\sin ax$ – 28 (b) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=xe^{2x}$ – 32

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\cos x$ – 33 (d) $\frac{d^2y}{dx^2}-y=xe^x\sin x$

5. (a) Find the orthogonal trajectories of the family of cardoid $r=a(1+\cos\theta)$. 54

(b) Using the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2}+n^2y=\sec nx$. 30

6. Solve any two of the following partial differential equations :

(a) $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ 70

(b) $(y+z)p+(z+x)q=x+y$ 69

(c) $(x^2-yz)p+(y^2-zx)q=z^2-xy$ 68

(d) $(y^2+z^2-x^2)p-2xyq=-2xz$ where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$ 72

7. Apply Charpit's method to find the complete integral any one of the following :

(a) $x^2p^2+y^2q^2=z^2$ 74

(b) $p^2+q^2-2px-2qy+1=0$ 77

8. (a) Prove that $P_n(x)=\frac{1}{2^n \lfloor n}\frac{d^n}{dx^n}(x^2-1)^n$. 94

(b) Prove that $\int_{-1}^{1}P_n(x)P_m(x)\,dx=0$ *if* $m\neq n$ 96

9. Prove the following relations for $J_n(x)$:

(a) $\frac{d}{dx}\{x^nJ_n(x)\}=x^nJ_{n-1}(x)$ 80

(b) $xJ_n'(x)=-nJ_n(x)+xJ_{n-1}(x)$ 80

10. (a) Find the Laplace transforms of t^3e^{-3t} 108

(b) Show that : $L(t\sin at)=\frac{2as}{(s^2+a^2)^2}$ 122

11. (a) Find the inverse transforms of any one of the following :

(i) $\dfrac{s^2-3s+4}{s^3}$ **– 113** (ii) $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$ **– 113**

(b) If L {f (t)} = $\bar{f}(s)]$, then show that L {t^n f (t)} = $(-1)^n$ where n = 1, 2, 3, ... **109**

12. Apply the method of Laplace transform to solve any one of the following differential equation :

(a) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+5y = e^{-t}\sin t,$ where y (0) = 0 and $y'(0)=1.$ **125**

(b) $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+3y=e^{-t},$ where y (0) = 0 and $y'(0)=1.$

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